

Gauging Maximal Supergravities

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Abstract: We review recent progress in the gauging of maximal supergravity theories.

1 Introduction

Gaugings are the only known supersymmetric deformations of maximal supergravity. They may originate in various ways from fluxes and branes in higher dimensions. A gauging is obtained by coupling the abelian vector fields, which arise in toroidally compactified eleven-dimensional or IIB supergravity, to charges assigned to the elementary fields. The resulting gauge group is encoded in these charges, but supersymmetry severely restricts the possible gauge groups.

Originally, gaugings of maximal supergravity theories were constructed for gauge groups whose existence could be inferred from a Kaluza-Klein interpretation. The first examples were $SO(8)$ in four dimensions [1] and $SO(5)$ in seven dimensions [2], related to S^7 and S^4 compactifications of eleven-dimensional supergravity, respectively, and later $SO(6)$ in five dimensions [3], related to the S^5 compactification of IIB supergravity. At the same time it was also demonstrated how noncompact versions of these orthogonal gauge groups and contractions thereof could lead to viable gaugings [4]. After these initial developments the subject lay dormant for quite some time, until the importance of anti-de Sitter spaces, which are the natural ground states for gauged supergravities, became apparent. In the last few years new gaugings were discovered and explored (see, e.g. [5]), motivated by the adS/CFT correspondence and by the study of flux compactifications. Therefore it is a timely question to reinvestigate these gaugings from a more general viewpoint.

In this paper we review recent progress in this direction [6, 7]. A central feature of the maximal supergravities is that the scalars parametrize a G/H symmetric space. Here, G is usually referred to as the ‘duality group’ and H coincides with the R -symmetry group. The scalar fields are then described in terms of a spacetime dependent element of G , denoted by $\mathcal{V}(x)$, which transforms under rigid G transformations from the left and under local H transformations from the right. Upon choosing a gauge, the group element \mathcal{V} becomes the coset representative of G/H . Because H is already realized as a local invariance (with composite gauge fields), a gauging of the supergravity theory must be effected by embedding the new gauge group associated with the elementary gauge fields into the duality group G . The embedding of this gauge group into G is described

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by an embedding tensor Θ_M^α , which we will introduce in the next section. What we want to stress here is the following. In the gauging the kinetic terms are modified by covariantizing spacetime derivatives and field strengths with respect to the gauge group, without altering their appearance in the Lagrangian. In the same way the masslike terms and the potential associated with the gauging are described by the so-called T -tensor [1], which appears in the Lagrangian and the supersymmetry transformations in a way that is independent of the specific gauge group. Of course, just as the covariant derivatives and the field strengths, the T -tensor depends on the embedding tensor, but this dependence is implicit.

2 The embedding tensor

As explained above, the central question is which gauge groups can be embedded into the duality group G such that the supersymmetry of the Lagrangian can be preserved. The latter requires to introduce additional terms to the action and the supersymmetry transformation rules, which take a uniform form in terms of the T -tensor. In this section we discuss the embedding of the gauge group.

The (abelian) vector fields A_μ^M transform in a representation of the duality group G with generators denoted by $(t_\alpha)_M^N$, so that $\delta A_\mu^M = -\Lambda^\alpha (t_\alpha)_N^M A_\mu^N$. The gauging is effected by introducing gauge group generators X_M , which couple to the gauge fields and depend linearly on the G -generators t_α , *i.e.*,

$$X_M = \Theta_M^\alpha t_\alpha, \quad (1)$$

so that the gauging is characterized by a real *embedding tensor* Θ_M^α . The fact that the X_M generate a group, implies that the embedding tensor satisfies the closure condition,

$$\Theta_M^\alpha \Theta_N^\beta f_{\alpha\beta}^\gamma = f_{MN}^P \Theta_P^\gamma, \quad (2)$$

where the $f_{\alpha\beta}^\gamma$ and f_{MN}^P are the structure constants of G and of the gauge group, respectively. This condition implies that Θ is invariant under the gauge group. It is possible to rewrite the right-hand side of (2) in terms of the generators t_α , so that

$$f_{\beta\gamma}^\alpha \Theta_M^\beta \Theta_N^\gamma - (t_\beta)_N^P \Theta_M^\beta \Theta_P^\alpha = 0. \quad (3)$$

In four spacetime dimensions, the situation is somewhat different as one is dealing with both electric and magnetic charges, which together constitute the fundamental representation of $G = E_{7(7)}$. Here the gauge fields do not transform under the full duality group, as the dual magnetic potentials are lacking in the Lagrangian. The embedding tensor must therefore vanish for magnetic charges. We return to this issue in sect. 4.

The T -tensor is the H -covariant, field-dependent, tensor defined by

$$T_M^\alpha[\Theta, \phi] t_\alpha = \mathcal{V}^{-1}_M{}^N \Theta_N^\alpha (\mathcal{V}^{-1} t_\alpha \mathcal{V}), \quad (4)$$

where \mathcal{V} denotes the coset representative of G/H . When treating the embedding tensor as a spurionic object that transforms under the duality group, the Lagrangian and transformation rules remain formally invariant under G . Under such a transformation Θ would transform as $\Theta_M^\alpha t_\alpha \rightarrow g_M^N \Theta_N^\alpha (g t_\alpha g^{-1})$, with $g \in G$. Of course, when freezing Θ_M^α to a constant, the G -invariance is broken.

Because the closure identity (3) is covariant with respect to G , it leads to a corresponding H -covariant constraint quadratic in the T -tensor. Subsequent considerations

d	G	H	T
7	SL(5)	USp(4)	$10 \times 24 = 10 + \underline{15} + 40 + 175$
6	SO(5, 5)	USp(4) \times USp(4)	$16 \times 45 = 16 + \underline{144} + 560$
5	E ₆₍₆₎	USp(8)	$27 \times 78 = 27 + \underline{351} + 1728$
4	E ₇₍₇₎	SU(8)	$56 \times 133 = 56 + \underline{912} + 6480$
3	E ₈₍₈₎	SO(16)	$248 \times 248 = \underline{1} + 248 + \underline{3875} + 27000 + 30380$

Table 1: Decomposition of the T -tensor in various dimensions for maximal supergravities in terms of irreducible representations of G. According to the representation constraint, only the underlined representations are allowed.

will show that there is also a constraint linear in the T -tensor. This tensor contains a number of H-covariant tensors conventionally denoted by A_1 , A_2 and A_3 , which appear in the fermionic masslike terms proportional to $\psi_\mu\psi_\nu$, $\psi_\mu\chi$ and $\chi\chi$, respectively. Here ψ_μ denotes the gravitini fields and χ the matter spinor fields, whose supersymmetry variations $\delta\psi_\mu$ and $\delta\chi$ acquire terms linear in A_1 and A_2 , respectively. In addition to the masslike terms the Lagrangian contains a scalar potential quadratic in A_1 and in A_2 . Given the fact that gaugings are the only possible supersymmetric deformations of maximal supergravity, the T -tensor cannot contain any other tensors beyond $A_{1,2,3}$. Because the fermion fields transform according to known representations of H, the representation content of $A_{1,2,3}$ is in fact determined uniquely.

Now we note that *every* variation of the coset representative can be expressed as a (possibly field-dependent) G-transformation acting on \mathcal{V} from the right. For example, a rigid duality transformation acting from the left, can be rewritten as a field-dependent transformation from the right,

$$\mathcal{V} \rightarrow \mathcal{V}' = g \mathcal{V} = \mathcal{V} \sigma^{-1}. \quad (5)$$

with $\sigma^{-1} = \mathcal{V}^{-1} g \mathcal{V} \in G$, but also a supersymmetry transformation can be written in this form. Consequently, these variations of \mathcal{V} induce the following transformation of the T -tensor,

$$T_M{}^\alpha t_\alpha \rightarrow T'_M{}^\alpha t_\alpha = \sigma_M{}^N T_N{}^\alpha (\sigma t_\alpha \sigma^{-1}). \quad (6)$$

This implies that the T -tensor must constitute a representation of G. Observe that this is *not* an invariance statement; rather it means that the T -tensor (irrespective of the choice for the corresponding embedding tensor) varies under supersymmetry or any other transformation in a way that can be written as a (possibly field-dependent) G-transformation. Note also that the transformation assignment of the embedding tensor and the T -tensor are opposite in view of the relationship between g and σ . Since we know that the T -tensor representation should be decomposable into the H-representations associated with $A_{1,2,3}$, we can almost uniquely identify the representation to which the T -tensor belongs. As it turns out this leads to a restricted G-representation for the T -tensor, and therefore for the embedding tensor. The result of this analysis is displayed in table 1 for spacetime dimensions $d = 3, \dots, 7$.

Hence, consistent gaugings are characterized by embedding tensors that satisfy two constraints, one quadratic and one linear in the embedding tensor. The quadratic constraint ensures that the embedding tensor defines a proper subgroup of the duality group. The linear constraint implies that the embedding tensor belongs to a specific representation of the duality group, so that the corresponding T -tensor matches precisely with the tensors A_1 , A_2 and A_3 that appear in the fermionic masslike terms in the Lagrangian.

3 Example: some gaugings in $d = 5$ dimensions

As an application, let us consider some of the gaugings in $d = 5$ maximal supergravity and search for viable gauge groups embedded in the $E_{6(6)}$ duality group. We first assume that the gauge group is a subgroup of the $SL(2, \mathbb{R}) \times SL(6, \mathbb{R})$ maximal subgroup of $E_{6(6)}$. According to table 1, the embedding tensor must belong to the $(\mathbf{27} \times \mathbf{78}) \cap \mathbf{351}$ representation. With respect to $SL(2, \mathbb{R}) \times SL(6, \mathbb{R})$, the vector gauge fields, the $E_{6(6)}$ generators and the embedding tensor decompose according to,

$$\begin{aligned} \overline{\mathbf{27}} &\rightarrow (\mathbf{1}, \overline{\mathbf{15}}) + (\mathbf{2}, \mathbf{6}), \\ \mathbf{78} &\rightarrow (\mathbf{1}, \mathbf{35}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{2}, \mathbf{20}), \\ \mathbf{351} &\rightarrow (\mathbf{1}, \mathbf{21}) + (\mathbf{3}, \mathbf{15}) + (\mathbf{2}, \overline{\mathbf{84}}) + (\mathbf{2}, \overline{\mathbf{6}}) + (\mathbf{1}, \mathbf{105}), \end{aligned} \quad (7)$$

respectively. The table below summarizes how the embedding tensor couples the vector fields to the generators,

	$(\mathbf{1}, \mathbf{15})$	$(\mathbf{2}, \overline{\mathbf{6}})$
$(\mathbf{1}, \mathbf{35})$	$(\mathbf{1}, \mathbf{21}) + (\mathbf{1}, \mathbf{105})$	$(\mathbf{2}, \overline{\mathbf{6}}) + (\mathbf{2}, \overline{\mathbf{84}})$
$(\mathbf{3}, \mathbf{1})$	$(\mathbf{3}, \mathbf{15})$	$(\mathbf{2}, \overline{\mathbf{6}})$
$(\mathbf{2}, \mathbf{20})$	$(\mathbf{2}, \overline{\mathbf{6}}) + (\mathbf{2}, \overline{\mathbf{84}})$	$(\mathbf{3}, \mathbf{15}) + (\mathbf{1}, \mathbf{105})$

(8)

Observe that the left column refers to the adjoint representation of $E_{6(6)}$ written with upper index α as it appears in Θ_M^α , whereas the decomposition of the gauge group generators (1) refers to the $E_{6(6)}$ generators t_α written with lower index α . The top line specifies the possible charges to which the gauge fields can couple, which transform in the conjugate representations as compared to the gauge fields. In view of the fact that all representations in the $\mathbf{351}$ appear with multiplicity 1, equivalent representations in the table must be identified. Because we assume that the gauge group is contained in $SL(2, \mathbb{R}) \times SL(6, \mathbb{R})$, there is only one possible representation assignment for the embedding tensor, namely it should belong to the $(\mathbf{1}, \mathbf{21})$ representation, which does not appear in the bottom row. Hence, only the vector fields transforming in the $(\mathbf{1}, \overline{\mathbf{15}})$ representation are involved in the gauging and couple to the generators in the adjoint representation of $SL(6, \mathbb{R})$. The vector fields in the $(\mathbf{2}, \mathbf{6})$ cannot participate in the gauging and must be dualized into charged massive antisymmetric tensor fields.

Because the embedding tensor belongs to the $(\mathbf{1}, \mathbf{21})$ representation, its nonzero components are parametrized in terms of a six-by-six symmetric tensor θ_{AB} according to $\Theta_{[AB]}^C{}_D = \delta_{[A}^C \theta_{B]D}$, where θ_{AB} is characterized by p eigenvalues $+1$, q eigenvalues -1 and $r = 6 - p - q$ eigenvalues 0 . The gauge group generators can be decomposed in terms of the $SL(6, \mathbb{R})$ generators $t_A{}^B$ as follows,

$$X_{AB} = \Theta_{[AB]}^C{}_D t_C^D = \theta_{D[A} t_{B]}^D. \quad (9)$$

These generators define the $CSO(p, q, r)$ subgroup of $SL(6, \mathbb{R})$, which has dimension $15 - \frac{1}{2}r(r-1)$ and leaves the embedding tensor invariant. The gaugings are thus completely determined and encoded in the 27 conjugacy classes of θ_{AB} leading to 15 inequivalent gaugings [3, 8]. Because we already established the closure of the gauge group, the quadratic constraint (3) on the embedding tensor does not give rise to additional restrictions.

A second application is based on the subgroup $SO(5, 5) \times SO(1, 1)$. This semisimple group is not a maximal subgroup of $E_{6(6)}$, but it becomes maximal upon including 16 additional nilpotent generators transforming in the $\mathbf{16}_{-3}$ representation. We consider gauge

groups that are a subgroup of this non-semisimple maximal subgroup. The decomposition of the relevant $E_{6(6)}$ representations with respect to $SO(5, 5) \times SO(1, 1)$ is given by,

$$\begin{aligned}\overline{\mathbf{27}} &= \overline{\mathbf{16}}_{-1} + \mathbf{10}_{+2} + \mathbf{1}_{-4}, \\ \mathbf{78} &= \mathbf{45}_0 + \mathbf{1}_0 + \mathbf{16}_{-3} + \overline{\mathbf{16}}_{+3}, \\ \mathbf{351} &= \mathbf{144}_{+1} + \mathbf{16}_{+1} + \mathbf{45}_{+4} + \mathbf{120}_{-2} + \mathbf{10}_{-2} + \overline{\mathbf{16}}_{-5}.\end{aligned}\tag{10}$$

The couplings induced by the embedding tensor are shown in the table below,

	$\mathbf{16}_{+1}$	$\mathbf{10}_{-2}$	$\mathbf{1}_{+4}$
$\mathbf{45}_0$	$\mathbf{144}_{+1} + \mathbf{16}_{+1}$	$\mathbf{10}_{-2} + \mathbf{120}_{-2}$	$\mathbf{45}_{+4}$
$\mathbf{1}_0$	$\mathbf{16}_{+1}$	$\mathbf{10}_{-2}$	
$\mathbf{16}_{-3}$	$\mathbf{120}_{-2} + \mathbf{10}_{-2}$	$\overline{\mathbf{16}}_{-5}$	$\mathbf{16}_{+1}$
$\overline{\mathbf{16}}_{+3}$	$\mathbf{45}_{+4}$	$\mathbf{144}_{+1} + \mathbf{16}_{+1}$	

(11)

Again equivalent representations in the embedding matrix should be identified as they appear with multiplicity one in the $\mathbf{351}$ representation. The generators transforming as the $\mathbf{16}_{-3}$ representation (denoted in the table above by the conjugate $\overline{\mathbf{16}}_{+3}$) cannot be involved in the gauging, as they do not belong to the maximal subgroup that we have selected. Therefore only two representations are allowed for the embedding tensor, namely $\mathbf{144}_{+1}$ and $\mathbf{45}_{+4}$. No gaugings have been worked out so far with embedding tensors that transform reducibly as $\mathbf{144} + \mathbf{45}$. The two irreducible cases can readily be identified and may originate from $d = 6$ dimensions. An embedding tensor belonging to the $\mathbf{144}_{+1}$ representation is induced by $d = 6$ gauged supergravity, as its embedding matrix must belong to the $\mathbf{144}$ representation of the $SO(5, 5)$ duality group (*c.f.* table 1), upon dimensional reduction on S^1 .

An embedding tensor belonging to the $\mathbf{45}_{+4}$ representation is obtained by a Scherk-Schwarz reduction from $d = 6$ dimensions, where ungauged maximal supergravity is invariant under $SO(5, 5)$ duality. Indeed, the representations of the vector fields are in accord with this interpretation. The embedding tensor is parametrized in terms of a matrix $\theta_p{}^q$, with $p, q = 1, \dots, 16$, belonging to the spinor representation of $SO(5, 5)$, such that

$$X_0 = \theta_p{}^q t^p{}_q, \quad X_p = \theta_p{}^q t_q, \tag{12}$$

where the $t^p{}_q$ are the generators of $SO(5, 5)$, while t_p are the generators belonging to the $\mathbf{16}_{-3}$ representation. This differs from the assignment in the table (11), because the embedding tensor has an upper index α associated with the Lie algebra of $E_{6(6)}$, whereas the generators carry lower indices. The gauge algebra obviously closes,

$$[X_0, X_p] = \theta_p{}^q X_q, \quad [X_p, X_q] = 0. \tag{13}$$

From (12) it follows that the null vectors of $\theta_p{}^q$ correspond to gauge fields that do not participate in the gauging and remain abelian. Obviously the maximal dimension of the gauge group is equal to 17. The vector fields in the $\mathbf{10}_{+2}$ are generically charged under X_0 and must be converted to charged tensor fields in order to be described in terms of a Lagrangian. Because the gauge group closes in view of (13), the quadratic constraint (3) must be satisfied, so that there are no further restrictions.

The above examples have consistent gauge groups, so there is no need to verify the quadratic constraint. This is no longer obvious when choosing the embedding tensor in a reducible representation, or when assuming that the gauge group is embedded in the

maximal compact subgroup generated by the generators of $\text{SO}(5, 5) \times \text{SO}(1, 1)$ combined with the generators transforming in the $\overline{\mathbf{16}}_{+3}$ representation. In this case there are many more options for the embedding tensor consistent with the representation constraint. A more complete analysis is far from trivial and should involve the quadratic constraint (3).

4 Subtleties in $d = 4$ dimensions

In four spacetime dimensions the Lagrangian is not invariant under $G = E_{7(7)}$, although the combined field equations and the Bianchi identities are. The Lagrangian is invariant under a subgroup $G_{\text{electric}} \subset E_{7(7)}$ and the gauge group has to be a subgroup of G_{electric} . The Lagrangian is not unique as there are many Lagrangians leading to equivalent field equations and Bianchi identities, each one with a corresponding invariance group G_{electric} . For any given Lagrangian gaugings can be studied along the lines presented earlier. As an example we consider the Lagrangian with $G_{\text{electric}} = \text{SL}(8, \mathbb{R})$. Hence we start with the branching rules under this group of the relevant representations,

$$\begin{aligned} \mathbf{56} &\rightarrow \mathbf{28} + \overline{\mathbf{28}}, \\ \mathbf{133} &\rightarrow \mathbf{63} + \mathbf{70}, \\ \mathbf{912} &\rightarrow \mathbf{36} + \mathbf{420} + \overline{\mathbf{36}} + \overline{\mathbf{420}}, \end{aligned} \tag{14}$$

where the $\mathbf{28}$ representation in the first branching corresponds to the gauge potentials and the conjugate $\overline{\mathbf{28}}$ corresponds to the dual magnetic potentials, which cannot be incorporated in the gauging. The branchings of products of the relevant representations (14) that belong to the $\mathbf{912}$, and thus identify acceptable representations of a T -tensor, is conveniently summarized by the table below,

	$\mathbf{28}$	$\overline{\mathbf{28}}$
$\mathbf{63}$	$\mathbf{36} + \mathbf{420}$	$\overline{\mathbf{36}} + \overline{\mathbf{420}}$
$\mathbf{70}$	$\overline{\mathbf{420}}$	$\mathbf{420}$

(15)

The $\mathbf{420}$ and the $\overline{\mathbf{420}}$ representations appear twice in the above table but have multiplicity 1 according to (14). Therefore their presence implies a coupling to both electric and magnetic charges, which is not permitted. That leaves an embedding tensor transforming in the $\overline{\mathbf{36}}$ representation as the only possibility. According to (15), the gauge group generators are then decomposable in the generators of $\text{SL}(8, \mathbb{R})$. Thus we conclude that all possible gaugings for the Lagrangian in the $\text{SL}(8, \mathbb{R})$ basis are encoded by an embedding matrix in the $\overline{\mathbf{36}}$ representation. The gauging is completely determined and encoded in the 44 nontrivial conjugacy classes of an eight-by-eight symmetric tensor transforming in the $\overline{\mathbf{36}}$ representation, characterized by its eigenvalues ± 1 or 0. These conjugacy classes correspond to 24 inequivalent gaugings with gauge group $\text{CSO}(p, q, r)$ (this time with $p + q + r = 8$) and dimension $28 - \frac{1}{2}r(r - 1)$. As we are dealing with a consistent gauge group, the quadratic constraint (3) becomes superfluous, as in the previous cases.

Because the $E_{7(7)}$ charges combine electric and magnetic ones, there is a new feature in this case. The requirement that the charges can all be chosen as electric ones (upon a suitable electric-magnetic duality transformation) so that the gauge group can be embedded into a subgroup G_{electric} of a certain Lagrangian, implies that the embedding tensor should satisfy an $E_{7(7)}$ -invariant quadratic constraint,

$$\Theta_M{}^\alpha \Theta_N{}^\beta \Omega^{MN} = 0, \tag{16}$$

where Ω^{MN} is the $E_{7(7)}$ invariant symplectic matrix. This constraint has been proven directly for the $d = 4$ T -tensor based on electric charge assignments [6], and it may be compared to the previous quadratic constraint (3). In order to do this, consider the symmetric product of two **912** representations,

$$(\mathbf{912} \times \mathbf{912})_{\text{symmetric}} = \mathbf{133} + \mathbf{8645} + \mathbf{1463} + \mathbf{152152} + \mathbf{253935} , \quad (17)$$

It turns out that the constraint (16) constitutes precisely the first two representations, **133** + **8645**. Furthermore, the constraint (3) constitutes the same representations, provided the embedding tensor is restricted to the **912** representation. Hence, (16) is therefore *not* independent. However, this situation leads to the observation that, when imposing the representation constraint that requires the embedding tensor to belong to the **912** representation, one can follow two different strategies. Either one keeps track of the assignments of electric and magnetic charges (as we did above), in which case the quadratic constraint (3) is automatically satisfied, or, one directly imposes the constraint (3), in which case it is guaranteed that there will exist a suitable Lagrangian (via electric-magnetic duality) such that the corresponding gauging can be switched on. The latter is the strategy that we follow in the application described below.

5 Gaugings from IIB fluxes

As a last example we consider gaugings of maximal supergravity in four spacetime dimensions that can in principle be generated by three- and five-form fluxes of the type-IIB theory. The proper setting is based on the decomposition of the $E_{7(7)}$ group according to $SL(2, \mathbb{R}) \times SL(6, \mathbb{R})$. The relevant embedding proceeds as follows,

$$E_{7(7)} \longrightarrow SL(6, \mathbb{R}) \times SL(3, \mathbb{R}) \longrightarrow SL(6, \mathbb{R}) \times SL(2, \mathbb{R}) \times SO(1, 1) . \quad (18)$$

There is another, inequivalent, embedding, but the one above is relevant for the IIB theory, with $SL(2, \mathbb{R})$ the S-duality group. Under this embedding, the **56** representation of electric and magnetic charges, and the adjoint representation of $E_{7(7)}$ decompose as follows,

$$\begin{aligned} \mathbf{56} &\rightarrow (\bar{\mathbf{6}}, \mathbf{1})_{-2} + (\mathbf{6}, \mathbf{2})_{-1} + (\mathbf{20}, \mathbf{1})_0 + (\bar{\mathbf{6}}, \mathbf{2})_{+1} + (\mathbf{6}, \mathbf{1})_{+2} , \\ \mathbf{133} &\rightarrow (\mathbf{1}, \mathbf{2})_{-3} + (\mathbf{15}, \mathbf{1})_{-2} + (\bar{\mathbf{15}}, \mathbf{2})_{-1} + (\mathbf{1}, \mathbf{1})_0 + (\mathbf{35}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0 \\ &\quad + (\mathbf{15}, \mathbf{2})_{+1} + (\bar{\mathbf{15}}, \mathbf{1})_{+2} + (\mathbf{1}, \mathbf{2})_{+3} . \end{aligned} \quad (19)$$

The embedding tensor, transforming in the **912** representation, decomposes into a large number of representations and among them are the $(\bar{\mathbf{6}}, \mathbf{1})_{+4}$ and $(\mathbf{20}, \mathbf{1})_{+3}$ representations that are potentially related to the five- and three-form fluxes of the IIB theory. Hence we investigate embedding tensors expressed in terms of two tensors, $\theta_{\Lambda\Sigma\Gamma}{}^\tau$ and θ^Λ , where $\Lambda, \Sigma, \dots = 1, \dots, 6$ and $\tau = 1, 2$ refer to $SL(6, \mathbb{R})$ and $SL(2, \mathbb{R})$ indices in the defining representations, respectively. These embedding tensors couple the gauge fields to the $E_{7(7)}$ generators belonging to the $(\bar{\mathbf{15}}, \mathbf{2})_{-1} + (\mathbf{15}, \mathbf{1})_{-2} + (\mathbf{1}, \mathbf{2})_{-3}$ representation, which we denote by $t^{\Lambda\Sigma\tau}$, $t_{\Lambda\Sigma}$ and t^τ , respectively. The generators of the gauge group are,

$$\begin{aligned} X_{\Lambda\Sigma\Gamma} &= 2\varepsilon_{\tau\sigma}\theta_{\Lambda\Sigma\Gamma}{}^\tau t^\sigma , \\ X^{\Lambda\tau} &= \frac{1}{6}\varepsilon^{\Lambda\Sigma\Gamma\Omega\Pi\Delta}\theta_{\Sigma\Gamma\Omega}{}^\tau t_{\Pi\Delta} + \theta^\Lambda t^\tau , \\ X_\Lambda &= \varepsilon_{\tau\sigma}\theta_{\Lambda\Sigma\Gamma}{}^\tau t^{\Sigma\Gamma\sigma} + \theta^\Sigma t_{\Lambda\Sigma} . \end{aligned} \quad (20)$$

The above result, which defines the embedding matrix Θ_M^α in terms of $\theta_{\Lambda\Sigma\Gamma}^\tau$ and θ^Λ , is uniquely determined by requiring that Θ_M^α is an element of the **912** representation. Note that there is a certain degeneracy in these definitions; not all generators X_M are linearly independent, and there are at most 20 independent generators. The quadratic constraint, which ensures the closure of the gauge generators, implies

$$\varepsilon^{\Lambda\Sigma\Gamma\Omega\Pi\Delta} \theta_{\Lambda\Sigma\Gamma}^\tau \theta_{\Omega\Pi\Delta}^\sigma = 0 . \quad (21)$$

The gauge algebra has the following non-vanishing commutation relations,

$$\begin{aligned} [X_\Lambda, X_\Sigma] &= -2\varepsilon_{\tau\sigma} \theta_{\Lambda\Sigma\Gamma}^\tau X^{\Gamma\sigma} - \theta^\Gamma X_{\Lambda\Sigma\Gamma} , \\ [X_\Lambda, X^{\Sigma\tau}] &= -\frac{1}{6}\varepsilon^{\Sigma\Gamma_1\Gamma_2\Gamma_3\Omega_1\Omega_2} \theta_{\Gamma_1\Gamma_2\Gamma_3}^\tau X_{\Lambda\Omega_1\Omega_2} . \end{aligned} \quad (22)$$

Without considering a specific $N = 8$ Lagrangian, we have thus constructed a novel gauging related to a possible IIB flux compactification. The T -tensor and the corresponding potential can be constructed by choosing a convenient representative for the $E_{7(7)}/SU(8)$ coset space. The details can be found in [7], where we also prove that gaugings with nilpotent charges lead to a positive potential. This potential has no stationary points, but for a certain choice of the embedding tensor one can find domain wall solutions that can be lifted to ten-dimensional IIB supergravity.

In the approach of this section, where we simply start from a choice of the embedding tensor, one avoids the subtleties associated with electric-magnetic duality. The emphasis is on finding an admissible embedding tensor, without the need of first constructing the full Lagrangian. The approach is equally well applicable to maximal supergravity in any other number of spacetime dimensions.

This work is partly supported by EU contract HPRN-CT-2000-00131.

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